STUDY OFSTRUCTURAL ISOMORPHISM AMONG MECHANISMS KINEMATIC CHAINS

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Abstract

In this paper, the adjacency and incidence matrices to determine the topological structure of a mechanism upto structural isomorphism are studied and found that these methodssatisfy the uniqueness and de-codability conditions but are computationally inefficient. The other methods of identification of structural isomorphisms including the characteristic polynomials, the MAX code, and the degree code, are also presented. The paper is extremely useful for P.G students, research scholar and designer of mechanisms at the conceptual stage of design.

Key words: Structural analysis, isomorphisms, kinematic graphs, contracted gaph, gear train.

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1. Introduction

Structural analysis means the study of the connection among the members of a mechanism kinematic chains and its mobility. Mainly, it is related with the fundamental relationships among the dof, number of links, number and the type of joints used in a mechanism. The structural analysis does not deal with the physical dimensions of the links. The structural analysis only deals with the general functional characteristics of a mechanism. Mostly, graph theory is used as a helping tool in the study of the kinematic structure of mechanisms. The on hand study focused only on mechanisms whose corresponding graphs are planar and contain no articulation points or bridges. A graph having a bridge means that the mechanism is a combination of two mechanisms connected in series with a common link but no common joint, or with a common joint but no common link. Such mechanisms are considered as two separate mechanisms. The topological structure of a mechanism kinematic chain is represented by a graph.

An important step in structural synthesis of kinematic chains and mechanisms is the identification of isomorphic structures. Undetected isomorphic structures lead to duplicate solutions, while falsely identified isomorphisms reduce the number of feasible solutions for new designs. Several methods of identification have been proposed.Some are based on visual approaches while others are based on heuristic approaches. Each method has its own advantages and disadvantages [1-23].Both the adjacency and incidence matrices determine the topological structure of a mechanism up to structural isomorphism. They satisfy the uniqueness and decodability conditions. However, they are computationally inefficient. For this reason, in this paper, other methods of identification have been proposed.

9. Structural Isomorphism

Two kinematic chains or mechanisms are said to be *isomorphic* if they share thesame topological structure. In terms of graphs, there exists a one-to-one correspondencebetween their vertices and edges that preserve the incidence. Mathematically,structural isomorphisms can be identified by their adjacency or incidence matrices.But, the form of an adjacency matrix is dependent on the labeling of links in akinematic chain.

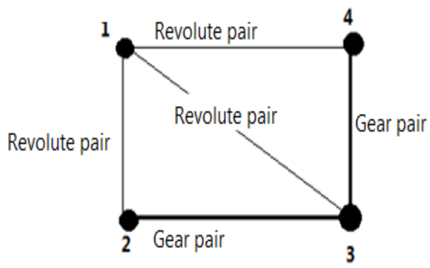
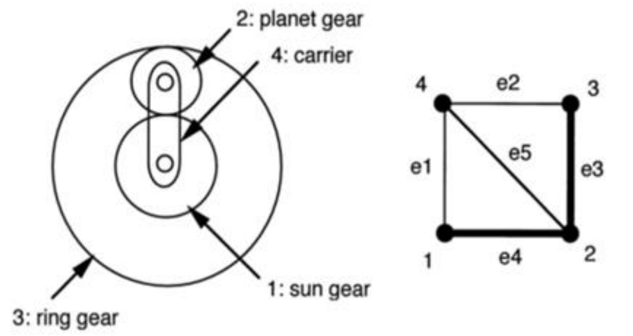
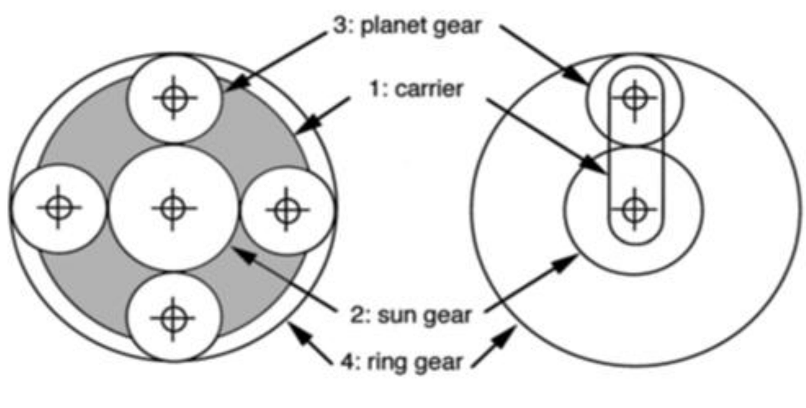


Figure 10:Graph representation of the planetary gear set shown in Figure 2



(a) Functional schematic (b) Graph representation

Figure 11: A planetary gear train and its kinematic graph



(a) Schematic diagram (b) Kinematic representation

FIGURE 12 :Schematic diagram and kinematic representation.

The graph in Figure 11 is obtained from a relabeling of thevertices of the graph in Figure 10. As a result, the adjacency matrix for figure 10 is given by equation (4) while the adjacency matrix for Figure 11is represented by eq.(5) .

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & g & 0 \\ 1 & g & 0 & g \\ 1 & 0 & g & 0 \end{bmatrix} \text{---------------(4)}$$

$$A^* = \begin{bmatrix} 0 & g & 0 & 1 \\ g & 0 & g & 1 \\ 0 & g & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \text{-----------------------------------} (5)$$

Although the graphs shown in Figures 10 and Figures 11 represent the same gear traingiven in Figure 12,their adjacency matrices do not assume the same form. The difference comes from thelabeling of the links. In fact, the two adjacency matrices, Equations (4) and (5),are related by a permutation of the rows and the corresponding columns.

If Sis a column matrix whose elements represent the labeling of the links of akinematic chain and S^*be another column matrix whose elements correspond to arelabeling of the links of the same kinematic chain. Then there exists a permutationmatrix, *P*, such that

$S^* = PS$ --------------------------------------(6)

Here ,$A^*$is related to A by a congruence transformation,

$A^* = P^T AP$ ---(7)

WhereP^Tdenotes the transpose of *P*. Theoretically, the permutation matrix *P* can bederived by reordering the columns of an identity matrix. It has a positive or negativeunit determinant and the transpose is equal to its inverse [17].

For example, if the column matrix for the graphs shown in Figure 10 is given by eq. (8) then the column matrix for the graph shown in Figure 11 will be represented by eq. (9). Therefore, the permutation matrix is given by eq. (10).

$$S = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{---------------------------------} (8)$$

$$S^* = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix} \text{-----------------------------} (9)$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{------------------------------------}(10)$$

clearly, Equations (8), (9), and (10) satisfy Equation (6). SubstitutingEquations (4) and (10) into Equation (7) yields

$$A^* = P^T AP = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & g & 0 \\ 1 & g & 0 & g \\ 1 & 0 & g & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & g & 0 & 1 \\ g & 0 & g & 1 \\ 0 & g & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \text{------------------------------------}(11)$$

Equations (6) and (7) constitute the definition of *structural isomorphism.* Inotherwords, two kinematic structures are said to be isomorphic if there exists a one-toonecorrespondence between the links of the two kinematic chains, Equation (6),and when the links are consistently renumbered, the adjacency matrices of the twokinematic chains become identical, Equation (7).

10. Permutation Group and Group of Automorphisms

Here the concept of a permutation groupof a graph is given.Automorphicgraphs are useful for elimination of isomorphic graphs at the outset.

Let there is a set of elements: a, b, c, d, e, and f . These elements may representthe vertices or edges of a graph, or the links or joints of a kinematic chain. Letthese elements be arranged in a reference sequence, say (a, b, c, d, e, f). We callan alternate sequence (b, c, a, d, f, e) a permutation of (a, b, c, d, e, f), in whicha → b (element a is mapped into b), b → c, c → a, d → d, e → f , and f → e.The reference sequence, (a, b, c, d, e, f), is called the identity permutation.

In a permutation, some elements may map into other elements, whereas others maymap into themselves. A mapping of the type a → b → c → a, denoted by (abc),is said to form a cycle. We define the length of a cycle by the number of elements inthat cycle. In particular, a cycle of length 1 maps an element into itself; that is, (d)means that d → d.

A permutation is said to be represented in cycles if each element occurs exactlyonce and the mapping of the elements is represented by the cycles. For example,the mapping of (a, b, c, d, e, f)into (b, c, a, d, f, e) has a cyclic representation of(abc)(d)(ef), where the lengths of the 3 cycles are 3, 1, and 2, respectively. Inparticular, the identity permutation is denoted by (a)(b)(c)(d)(e)(f).

10.1 Group

A set of *n* elements, *a*1, *a*2, . . . , *an*, is said to form a *group* under a given *groupoperation*, denoted by the multiplication symbol *ai· aj*, if the following axioms aresatisfied [7]:

1. *Closure:* If $a_i$and $a_j$are two elements of the group, then $a_i \cdot a_j$is also anelement of the group.
2. *Associativity:* For all elements of the group,

$(a_i \cdot a_j) \cdot a_k = a_i \cdot (a_j \cdot a_k)$

In this regard, $(a_i \cdot a_j) \cdot a_k$is denoted unambiguously by $a_i \cdot a_j \cdot a_k$.

3. *Existence of an identity element:* There exists an element, a_{id}, such that

$a_{id} \cdot a_j = a_j \cdot a_{id} = a_j$

for all elements of the group.

4. *Existence of inverses:* For each element, a_j, there exists an *inverse element,*

$a^{-1}{}_j$, such that

$aj \cdot a^{-1}{}_j = a_{id}$.

We note that the group operation is not necessarily commutative; that is $a_j \cdot a_k \neq a_k \cdot a_j$.

A *permutation group* is a group whose elements are permutations. The *groupoperation* for a permutation group is defined as follows. Let permutation $a_j$mapelement $x_p$into x_q, whereas permutation $a_k$maps element $x_q$into x_r. Then theproduct $a_j \cdot a_k$maps $x_p$into x_r.

10.2 Group of Automorphisms

Considering *labeled* graphwhen its vertices are labeled by the integers 1, 2, . . . ,*n*. In this regard, a labeled graph is mapped into another labeled graph when the *n*integers are permuted. For some permutations, a labeled graph may map into itself.The set of those permutations which map the graph into itself form a group called a*group of automorphisms*. This group of automorphisms is said to be a *vertex-induced*group [9].Similarly, the edges of a graph may be labeled. We call the group of permutationsthat maps the graph into itself an *edge-induced* group of automorphisms.

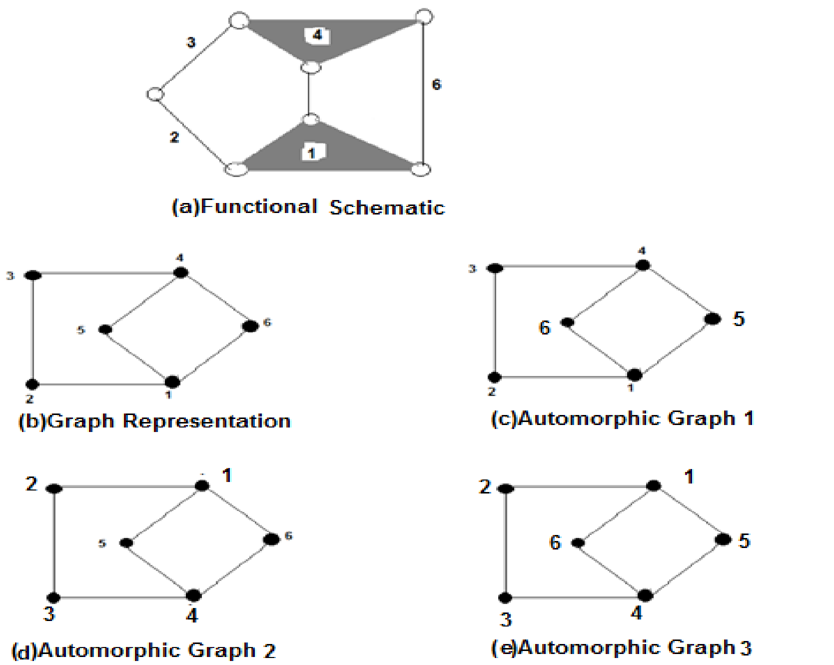


Figure 8:----Stephenson Chain, its Graph Representation,and Automorphic Graph

11. Identification of Structural Isomorphism

11.1 Identification of Structural Isomorphism by Classification

Kinematic chains (or graphs) can be classified into families according to the numberof links, number of joints, various link assortments, etc. Obviously, kinematic chainsof different families cannot be isomorphic with one another. This fact has been usedfor classification and identification of the topological structures of kinematic chains.

For example, Buchsbaum and Freudenstein [6] classified the graphs of epicyclicgear trains according to their (1) number of vertices, (2) number of edges, and (3)*vertex degree listing.* Yan and Hwang [20, 21] expanded the above classificationmethod to include other attributes such as joint assortments, and so on.

Kinematic chains can also be classified by their corresponding contracted graphs.Obviously, two kinematic chains that belong to two different contracted graphs cannotbe isomorphic. For example, Figure 13 shows two (11, 14) graphs. Both graphscontain 11 vertices and 14 edges. Also , they share the same vertex degreelisting of 6410. Hence, up to this level of classification, it seems that they are isomorphic. But, the contracted graph of the graph shown in Figure 13(a) has been shown in Figure 14 (a), whereas the contracted graph of the graph shown in Figure

13(b) has been shown in Figure 14 (b). Since both the contracted graphs in Figure 14(a) and 14(b) are different, so, they are not isomorphic.

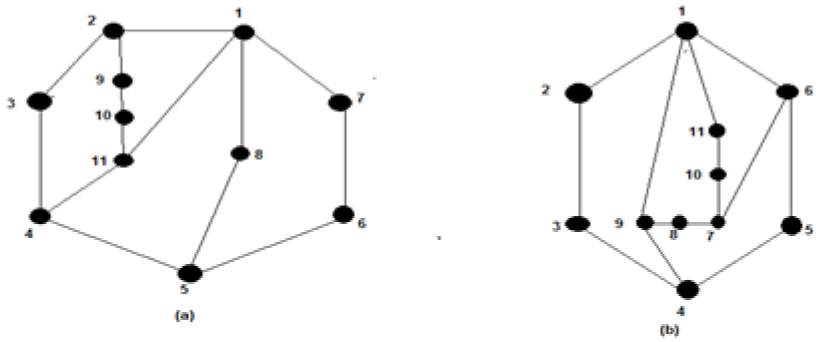


Figure 13: Two (11,14) Graph

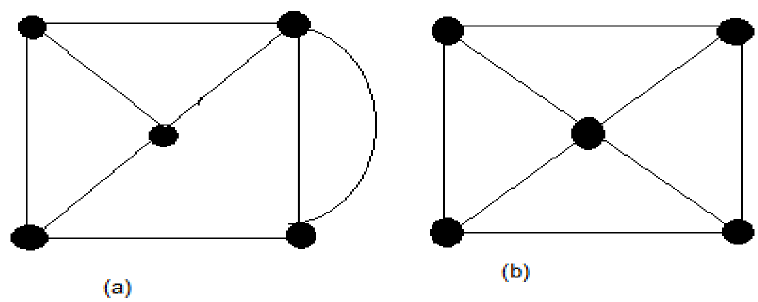


Figure14: Contracted graph of the graph shown in Figure 13 (a)and (b) respectively

11.2 Identification of Structural Isomorphism by Characteristic Polynomial

We know that the problem of testing structural isomorphismis equivalent to one of determining a permutation matrix *P* that transformsthe *A* into *A*∗for the two kinematic chains in question. For an *n*-link kinematic chain,there are *n*! possible ways of labeling the links and, therefore, *n*! possible permutationmatrices. Therefore, it is impractical to identify the permutation matrix by trial-anderror.Fortunately, there exists a convenient method for the determination of such apermutation matrix.

A well-known theorem of matrix algebra states that the congruence relation givenby Equation (7) can exist only if the characteristic polynomials of the two adjacencymatrices, *A* and *A*∗, are equal to each other; that is,

$$|xI - A| = |xI - A^*| \quad \text{------------------------------------}(12)$$

holds for all *x*, where *x* is a dummy variable and *I* is an identity matrix of the sameorder as *A*. We conclude that

Theorem 1

The adjacency matrices of two isomorphic kinematic chains possess the same characteristicpolynomial.

The linkage characteristic polynomial for the graph shown in Figure10 is given by eq.(13) and the linkage characteristic polynomial for the graph shown in Figure 11 is given by eq.(14).

$$p(x) = \begin{vmatrix} x & -1 & -1 & -1 \\ -1 & x & -g & 0 \\ -1 & -g & x & -g \\ -1 & 0 & -g & x \end{vmatrix} = x4 - (3 + 2g^2)x^2 - 4gx \quad \text{------------------------------(13)}$$

$$p^*(x) = \begin{vmatrix} x & -g & 0 & -1 \\ -g & x & -g & -1 \\ 0 & -g & x & -1 \\ -1 & -1 & -1 & x \end{vmatrix} = x4 - (3 + 2g^2)x^2 - 4gx \quad \text{----------(14)}$$

Since p(x) = p*(x)*, the two graphs are most likely isomorphic.

The above theorem is a necessary, but not a sufficient conditionfor two kinematic chains to be isomorphic. Although this condition is not completelydiscriminatory, it can successfully distinguish the kinematic chains with up to eight links. Asthe number of links increases, however, the probability of failing to detect structuralisomorphism increases. Counter examples have been found where two nonisomorphickinematic chains share the same characteristic polynomial [14].

Figure 15 shows two (10, 13) nonisomorphic graphs sharing thecharacteristic polynomial:

$p(x) = x^{10} - 13x^8 + 53x^6 - 8x^5 - 82x^4 + 26x^3 + 39x^2 - 16x$ ------------------(15)

Similarly, Figure 16 shows two (11, 14) nonisomorphic graphs sharing the characteristicpolynomial:

$p(x) = x^{11} - 14x^9 + 65x^7 - 13x^5 + 112x^3 + 32x$ ---------------------------(16)

This method of identification needs the derivation of characteristic polynomials.Uicker and Raicu [17] presented a computer method for derivation of the coefficientsnumerically. Yan and Hall [18, 19] developed a set of rules for determination ofthe polynomials by inspection. Tsai [16] suggested the use of the random numbertechnique to improve the computational efficiency.

Because the characteristic polynomial cannot fully identify structural isomorphisms,the method is often augmented by other techniques such as classification ofkinematic chains according to their contracted graphs. For example, the graph shownin Figure 15(a) belongs to the contracted graph shown in Figure 14 (a), whereas the one shown in Figure 15(b) belongs to the Figure 14 (b) contracted graph. Therefore, these two graphs cannot be isomorphic although they share a common characteristic polynomial. Similarly, the graphs shown in Figure 16(a) and Figure 16(b) cannot be isomorphic because they belong to two different contracted graphs shown in Figure 14(a) and Figure 14 (b) respectively.

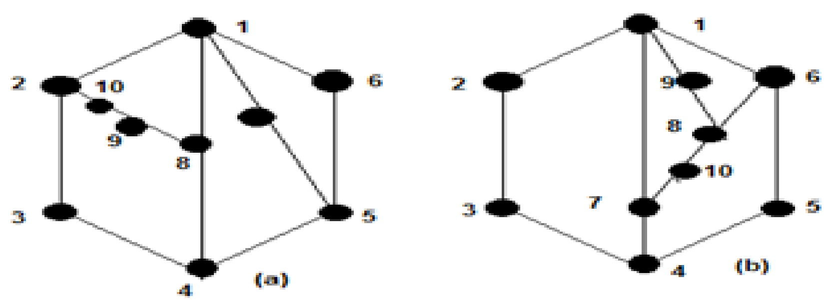


Figure 15:Two (10,13) nonisomorphic graphs

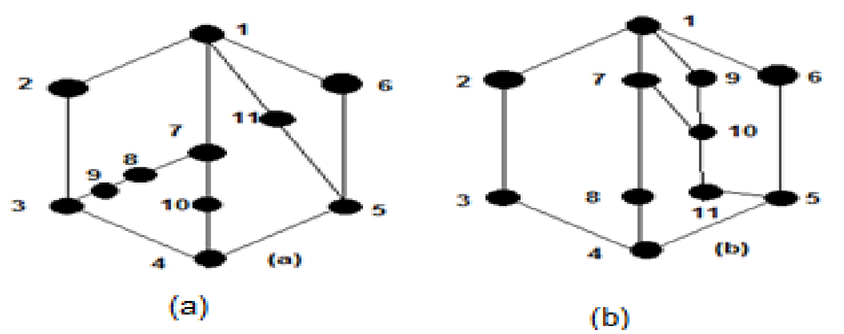


Figure 16: Two (11,14) nonisomorphic graphs

11.3 Identification of Structural Isomorphism by Optimum Code

Despite the extraordinary degree of discrimination, the method of characteristicpolynomial encounters difficulties in three respects. (1) The method is not decodable.(2) It is not a sufficient condition for identification of structural isomorphism.(3) The computational efficiency is poor. To overcome these difficulties, Ambekarand Agrawal [1, 2, 3] suggested a method of identification called the *optimum code*.In contrast to the characteristic polynomial method, the optimum code guarantees thedecodability and positive identification of structural isomorphism. The method involvesa technique for labeling the links of a kinematic chain such that a binary stringobtained by concatenating the upper triangular elements of the adjacency matrix rowby row, excluding the diagonal elements, is maximized. This is called the *MAX code*.To illustrate the concept, we consider the six-link Stephenson chain shown inFigure8 (a).The corresponding labeled graph is shown in Figure 8 (b). Let thelabeling of the vertices shown in Figure 8 (b) be denoted by an identity permutation$a_1 = (1)(2)(3)(4)(5)(6)$. Then the adjacency matrix is given by eq. (17).

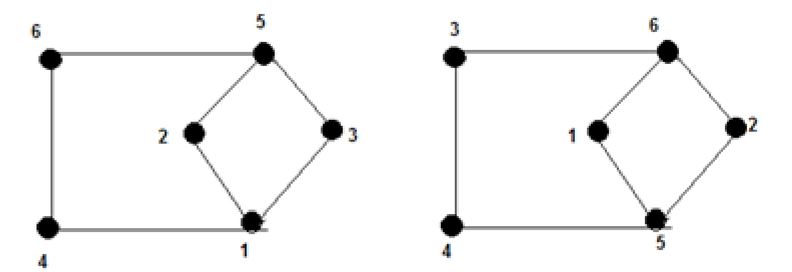
$$A1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \text{----------------------------------}(17)$$

Excluding the diagonal elements, there are 15 binary elements in the upper triangularadjacency matrix, namely 10011, 1000, 100, 11, and 0. Writing these elements insequence we obtain a binary string of 100111000100110, which can be convertedinto a decimal number as follows.

1001110001001102 = 214 + 211 + 210 + 29 + 25 + 22 + 21 = 20006 .

Figure 17(a) shows a different labeling of the vertices that corresponds to thepermutation $a2 = (1)(245)(36)$. For this labeling, the adjacency matrix becomes eq. (18).

$$A2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \text{-----------------------}(18)$$



(a) Graph labelled for MAX code (b) Graph Labelled for MIN code

Figure 17: Stephenson chain labeled for the MAX and MIN codes

Therefore, the upper triangular elements of*A*2 form a binary string of 111000010010011, which is equal to a decimal number of 28819.It can be shown that, among all possible labelings of the graph, the labeling shownin Figure 17(a) leads to a maximum number. We call the number 28819 the *MAXcode* of the Stephenson chain. We note that several labelings of a graph may lead tothe same MAX code due to the existence of graph automorphisms.Alternately,we can also search for a labeling of the Stephenson chain that minimizesthe binary string of the upper triangular elements. We call the resulting decimalnumber the *MIN code.* Intuitively, for the permutation $a3 = (15)\ (3)\ (246)$ shownin Figure 17(b), we obtain a minimum binary string of 000110011101100, whichgives 3308 as the MIN code.

Using the above method, the problem of testing structural isomorphism is convertedinto a problem of comparing the optimum codes of two kinematic chains in question.For an *n*-link kinematic chain, the method requires *n*! permutations to arrive at theoptimum code. Clearly, there is a need to develop a more efficient heuristic algorithmfor determination of the optimum code [1]. A poorly developed algorithm may leadto a local optimum and, therefore, may reduce the robustness of the method. From thedefinition of the MAX code, we observe that the first few rows of the upper triangularadjacency matrix constitute the most significant bits of the code. Furthermore, ineach row, the closer an element is to the diagonal of the adjacency matrix, the morecontribution it makes to the binary code. Therefore, any efficient algorithm shouldaim at shifting as many 1s to the most significant bits of the binary code as possible.Further, the concept of a group of automorphisms can be employed to further reducethe number of permutations.

11.4 Identification of Structural Isomorphism by Degree Code

In this section we describe a heuristic algorithm called the *degree code* [15]. Recallthat the degree of a vertex is defined as the number of edges incident to it. From thekinematics point of view, the degree of a vertex represents the number of joints on alink. Hence, a vertex of degree 2 denotes a binary link, a vertex of degree 3 representsa ternary link, and so on. In the degree code, the vertex degrees are used as a constraintfor labeling the links of a kinematic chain. Links of the same degrees are groupedtogether and the various groups of links are arranged in a descending order accordingto their vertex degree. While searching for an optimum code, permutations of the links are constrained within each group such that the degrees of all vertices are always keptin a descending order. This method not only preserves the advantages of the optimumcode, but also reduces the number of permutations needed for searching the optimum.

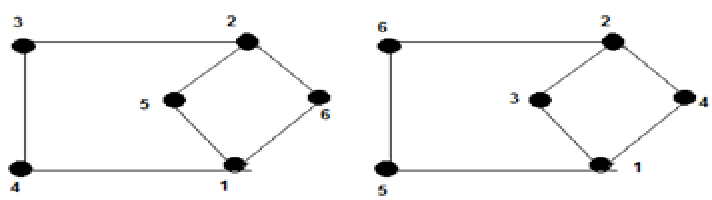
For example, if the links of an n-link kinematic chain are divided into 3 groups having$p$, q, and r number of links, respectively, where $n = p + q + r$, the total number ofpermutations reduces from $n!$ to$p!q!r!$.

The procedure for finding the degree code of a kinematic chain can be summarizedas follows:

1. Identify the degree of each vertex in the graph of a kinematic chain and arrangethe vertices of the same degree into groups.
2. Renumber the vertices according to the descending order of vertex degrees.
3. Permute the vertices of the same degree to get a new labeling of the graph.Similar vertices, if any, can be arranged in a subgroup to further reduce thenumber of permutations.
4. For each permutation, calculate the decimal number of the binary string obtainedby concatenating, row-by-row, the upper-right triangular elements of thecorresponding adjacency matrix.
5. The maximum number obtained from all possible permutations is defined asthe degree code.

For example, the degrees of vertices 1 through 6 of the graph shown in Figure 8(b)are 3, 2, 2, 3, 2, and 2, respectively. Since there are two vertices of degree 3 and fourvertices of degree 2, the vertices are divided into two groups: (1, 4) and (2, 3, 5, 6).As a first attempt, we relabel the graph as shown in Figure 18(a) where integers 1and 2 are assigned to the two vertices of degree 3. Under the new labeling, the twogroups consist of (1, 2) and (3, 4, 5, 6). We notice that vertices 1 and 2 are similar.Hence, there is no need to permute these two vertices. Similarly, there is no needto permute vertices 5 and 6. Following the above procedure, it can be shown thatamong all possible permutations, the permutation shown in Figure 18(b) produces amaximum number. The adjacency matrix is

$$A4 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \text{-----------------------------}(19)$$



(a) First relabeling (b) Degree code

Figure 18: Stephenson chain labeled for the degree code

Therefore, the binary string is 011101101000001, which is equal to a degree codeof 15169. We note that the degree code is smaller than the *MAX code*, becausepermutations of the links for the degree code are confined within each group ofvertices of the same degree.

Finally, wenotethat thedegreecodes for thegraphs showninFigures 15(a) and Figures 15(b) are28055807549442 and 28055872017416, respectively. Although the two graphs sharea common characteristic polynomial, their degree codes are unequivocally different.

12. Illustrative examples

12.1 Example 1: Symmetric Group of Three Elements

Let three elements, denoted by the integers 1, 2, and 3, be ordered in a reference sequence (1, 2, 3). We show that the following six permutations form a group:

Element	Permutation	Cyclic Representation
a1	(1, 2, 3) → (1, 2, 3)	(1)(2)(3)
a2	(1, 2, 3) → (1, 3, 2)	(1)(23)
a3	(1, 2, 3) → (2, 1, 3)	(12)(3)
a4	(1, 2, 3) → (2, 3, 1)	(123)
a5	a5 (1, 2, 3) → (3, 1, 2)	(132)
a6	a6 (1, 2, 3) → (3, 2, 1)	(13)(2)

Following the definition of group operation, we can construct a multiplication table:

Following the definition of group operation, we can construct a multiplication table:

	*a*1	*a*2	*a*3	*a*4	*a*5	*a*6
*a*1	*a*1	*a*2	*a*3	*a*4	*a*5	*a*6
*a*2	*a*2	*a*1	*a*5	*a*6	*a*3	*a*4
*a*3	*a*3	*a*4	*a*1	*a*2	*a*6	*a*5
*a*4	*a*4	*a*3	*a*6	*a*5	*a*1	*a*2
*a*5	*a*5	*a*6	*a*2	*a*1	*a*4	*a*3
*a*6	*a*6	*a*5	*a*4	*a*3	*a*2	*a*1

We conclude that every product is an element of the group; the associative law holds; *a*1 is the identity element; *a*4 and *a*5 are mutually the inverse of each other; and every other element is its own inverse.

12.2 Example 2:Group of Automorphisms of Stephenson Chain

Consider the Stephenson chain shown in Figure 8(a), where the six links are labeled from 1 to 6. The corresponding graph is shown in Figure 8(b). Let this labeling of the graph be the identity permutation, *a*1 = *(1)(2)(3)(4)(5)(6)*. Figures 8(c) through e show three permutations of the labeling that can be denoted as *a*2 =(1)(2)(3)(4)(56), *a*3 =(14)(23)(5)(6), and *a*4 =(14)(23)(56), respectively. In the following we show that the above four labeled graphs form a group of automorphisms.

Following the definition of group operation, we can construct a multiplication table:

	*a*1	*a*2	*a*3	*a*4
*a*1	*a*1	*a*2	*a*3	*a*4

$a2$	$a2$	$a1$	$a4$	$a3$
$a3$	$a3$	$a4$	$a1$	$a2$
$a4$	$a4$	$a3$	$a2$	$a1$

We conclude that every product is an element of the group; the associative law holds; $a1$ is the identity element; and every element is its own inverse. Therefore,$a1$, $a2$, $a3$, and $a4$ form a group of automorphisms.

Two vertices of a graph are said to be *similar* if they are contained in the same cycle of a permutation of a vertex-induced group of automorphisms. In the above example, vertices 1 and 4 are similar. Vertices 5 and 6, and 2 and 3 are also similar. Similar vertices have the same vertex degrees and their adjacent vertices also have the same vertex degrees. In other words, similar vertices possess the same attributes. Automorphic graphs are by definition isomorphic. Analogously, two edges of a graph are said to be *similar* it they are contained in the same cycle of a permutation of an edge-induced group of automorphisms.

13. Conclusions

The basic concept of graph theory is essential for structural analysis and synthesis of mechanisms kinematic chains. The topological structure of a kinematic chain can be represented by kinematic graph easily with the help of graph theory. The Grubler equation for determination of dof does not consider the concept of redundant dof. Although, the redundant dof has no role in torque transfer from input to output link but the presion and accuracy of design depends upon it (see Table 1). There are many mechanisms (over-constrained mechanisms) those do not obey Grublercriterion.Several methods of identification of structural isomorphism among mechanisms kinematic chains like identification by- classification, characteristic polynomials, optimum code and degree code has been studied.These structural characteristics are extremely useful for the development of computer algorithms for systematic enumeration of mechanisms. It is suggested that a designer should use the concept of contracted graphs to determine the structural isomorphism as it is a reliable, easy to compute method. The study is extremely helpful for the U.G. /P.G. students, research scholars and designers in their early age of learning at the conceptual stage of design.

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